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# A three-phase balancing unit for single-phase variable loads

Aroyan, George Francis; Lakey, Keith Gilbert

Massachusetts Institute of Technology

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**A THREE-PHASE BALANCING UNIT FOR  
SINGLE-PHASE VARIABLE LOADS**



**GEORGE FRANCIS AROYAN  
KEITH GILBERT LAKEY**

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A THREE-PHASE BALANCING UNIT  
FOR SINGLE-PHASE VARIABLE LOADS

by

GEORGE FRANCIS AROYAN

B.S., U. S. Naval Academy  
(1945)

KEITH GILBERT LAKEX

B.S., U. S. Naval Academy  
(1946)

SUBMITTED IN PARTIAL FULFILLMENT OF THE  
REQUIREMENTS FOR THE DEGREE OF  
NAVAL ENGINEER

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
(1951)



A THREE-PHASE BALANCING UNIT  
FOR SINGLE-PHASE VARIABLE LOADS

by

George Francis Aroyan and Keith Gilbert Lakey

Submitted for the degree of Naval Engineer in the  
Department of Naval Architecture and Marine Engineering on May 18, 1951.

ABSTRACT

The objective of this thesis is to show that an automatic balancing unit can be designed to reflect balanced three-phase currents from single-phase loads of variable magnitude and power factor. Such a balancing unit was designed, constructed, and successfully operated. The method employed was based upon static schemes which utilize a tapped reactor and capacitor. Feedback loops were designed to vary these balancing reactances in such a way that, for variable single-phase loads, the reflected three-phase currents were balanced and the three-phase power factor was improved to .866.

It is considered that the success of this investigation warrants further study and engineering development to attain the ultimate objectives of (1) complete elimination of all moving parts and (2) adaptation of this scheme for single-phase to three-phase power conversion.



Cambridge, Massachusetts  
May 18, 1951

Professor J. S. Newell  
Secretary of the Faculty  
Massachusetts Institute of Technology  
Cambridge, Massachusetts

Dear Sir:

In accordance with the requirements for the degree of Naval Engineer, a thesis entitled "A Three-Phase Balancing Unit for Single-Phase Variable Loads" is herewith submitted.

Respectfully,



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## INTRODUCTION

This thesis deals with a method of automatically reflecting a single-phase load so that it appears as a balanced three-phase load to the power supply.

A balancing unit has been designed which will accomplish this purpose with a minimum number of moving parts and without the use of rotating machinery. Such a unit would have several applications in naval electrical installations.\* On submarines in particular it is desired to eliminate rotating equipment for several reasons, the most important being reduction of noise, maintenance, space, and weight.

Lighting, interior communications, radio, radar, fire-control systems, and similar loads on naval vessels may produce an unbalance in the loads on the three phases of a-c ship's service generators. By means of the method of symmetrical components it can be shown that unbalanced phase currents in a generator will produce unbalanced terminal voltages which can in turn be resolved into a set of balanced positive and negative sequence components. The effects of this unbalance may be excessive heating of the generator itself or local "hot spots" in the windings of induction motors. Thus, if motor capacity is limited by the hottest phase of the stator, a 5 per cent unbalance in the applied potentials may reduce its capacity by 20 to 30 per cent.\*\*

The problem of controlling the effects of unbalanced single-phase loads has led to the development of several types of phase-balancing

---

\* See Appendix A.

\*\* Reference (1.), p. 287.



equipment. Rotating balancers tend to balance the voltages and currents on a power system by periodically absorbing and restoring energy to the system using in this process the energy stored by the inertia of moving parts. The most important rotating phase-balancers include the negative-sequence emf type, the series impedance type, and the shunt impedance type with series capacitor.\*

The preferable method of phase-balancing is to employ some sort of static network. Reference (3.) illustrates several transformer connections which only partly accomplish the purpose. Other schemes for improving the balance of systems by static means utilize unsymmetrical transformer taps or unsymmetrical transformer voltages to balance fixed single-phase loads.\*\* However, none of the above methods eliminate the negative sequence currents.

The method of phase-balancing selected for the purpose of this investigation consists of a tapped reactor and capacitor. This method was selected because it lends itself most readily to automatic control for balancing single-phase loads of variable magnitude and power factor.

The use of such a scheme was suggested by two applications involving a constant load. Reference (4.) describes a unit consisting of a tapped reactor and capacitor which was designed to balance a large single-phase load to three-phase aircraft inverters. Reference (5.) describes a similar unit developed in Great Britain for use with electric welding machines. In both of the above applications, the unit

---

\* Reference (2.), pp. 382-386.

\*\* Ibid.



was designed to balance a particular single-phase load; and any departure from this loading resulted in unbalance of the system.

It is the object of this thesis, therefore, to investigate the possibility of automatically changing the inductance and capacitance of such a unit so that balanced conditions are always reflected from a single-phase load of varying magnitude and power factor.

$$V_p \cos \theta = V_s \cos \theta_s \quad (1)$$

$$V_p \sin \theta = V_s \sin \theta_s \quad (2)$$

$$V_p \tan \theta = V_s \tan \theta_s \quad (3)$$

From equation (1):

$$V_p \cos \theta = V_s \cos \theta_s$$

It is assumed that the load is a pure resistance, and that the source is a pure voltage source. The power factor of the load is assumed to be unity, and the power factor of the source is assumed to be unity. The power factor of the load is assumed to be unity, and the power factor of the source is assumed to be unity.

$$V_p \cos \theta = V_s \cos \theta_s$$

It is assumed that the load is a pure resistance, and that the source is a pure voltage source. The power factor of the load is assumed to be unity, and the power factor of the source is assumed to be unity. The power factor of the load is assumed to be unity, and the power factor of the source is assumed to be unity.



## PROCEDURE

### EXPLANATION OF STATIC BALANCING METHOD

In order to lend significance to the various steps in this investigation, it first becomes necessary to show how a tapped reactor and capacitor can be used to balance a static load. Figure I shows the schematic diagram of such a static balancing unit, and Figure II shows the corresponding vector diagram drawn to represent balanced conditions. The single-phase load is connected across terminals B and C of the three-phase system. The reactor is connected across terminals A and B and is tapped through the capacitor to terminal C. Phase rotation is assumed A-B-C. From Figure I:

$$\bar{I}_{\phi A} = -\bar{I}_M - \bar{I}_{C1} \quad (1)$$

$$\bar{I}_{\phi B} = \bar{I}_M - \bar{I}_L - \bar{I}_{C2} \quad (2)$$

$$\bar{I}_{\phi C} = \bar{I}_L + \bar{I}_C \quad (3)$$

Under balanced conditions:

$$\bar{I}_{\phi A} = \bar{I}_{\phi B} \angle 120^\circ = \bar{I}_{\phi C} \angle 240^\circ$$

If we assume no losses in the balancing unit and the phase currents in phase with the line-to-line voltages, the power into the balancing unit must equal the power absorbed by the single-phase load. Referring to Figure II, we can write:

$$3E_\phi I_\phi \cos 30^\circ = E_L I_L \cos \theta^*$$

---

\*  $E_\phi$  and  $I_\phi$  represent the line-to-neutral voltage and the line current of a Y-connected three-phase system.  $E_L$  and  $I_L$  represent the load voltage and current. Note that  $E_L$  is thus the line-to-line voltage of the three-phase system.

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8. J. von Neumann, *Ann. of Math.* **43**, 317 (1945).
9. J. von Neumann, *Ann. of Math.* **43**, 337 (1945).
10. J. von Neumann, *Ann. of Math.* **43**, 357 (1945).

- (1)  $\vec{p}^2 = \vec{p}^2 + \vec{p}^2 = 2\vec{p}^2$
- (2)  $\vec{p}^2 = \vec{p}^2 + \vec{p}^2 = 2\vec{p}^2$
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THEORETICAL REFERENCES

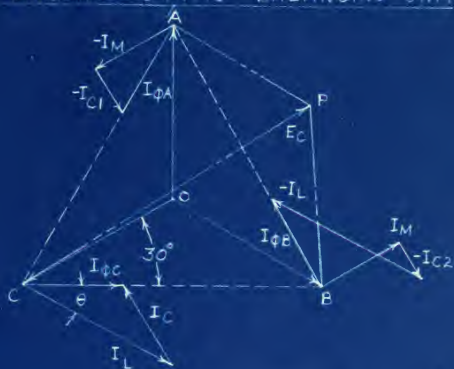
FIGURE I

SCHEMATIC DIAGRAM - STATIC BALANCING UNIT



FIGURE II

VECTOR DIAGRAM - STATIC BALANCING UNIT



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M.L.  
SFA



Since  $\sqrt{3} E_{\phi} = E_L :$

$$\sqrt{3} E_L I_{\phi} \cdot \frac{\sqrt{3}}{2} = E_L I_L \cos \theta$$

and 
$$I_{\phi} = \frac{2}{3} I_L \cos \theta \quad (4)$$

The capacitor current,  $I_C$ , is assumed leading the voltage  $E_C$  by  $90^\circ$ ; while the magnetising current,  $I_M$ , is assumed to lag the voltage across the reactor by  $90^\circ$ . By use of the superposition principles, the net current through the reactor can be considered as being made up of the magnetising current, a part of the capacitor current,  $I_{C2}$ , flowing between the reactor tap and terminal B, and the remainder of the capacitor current,  $I_{C1}$ , flowing between the tap and terminal A.

### GRAPHICAL STATIC ANALYSIS

The value of the capacitor and the reactor (and its tap position) needed to balance any single-phase load was obtained by drawing vector diagrams similar to Figure II for various assumed load conditions. In the remainder of this report, the tapped reactor will be considered as two independent reactors, e.g. the reactor connected between terminal A and the point P will be designated  $X_{L1}$  while that connected between terminal B and point P will be  $X_{L2}$ . It can be seen that the following equations apply:

$$j X_{L1} = \frac{\bar{V}_{AP}}{\bar{I}_M + \bar{I}_{C1}} \quad (5)$$

$$j X_{L2} = \frac{\bar{V}_{PB}}{\bar{I}_M - \bar{I}_{C2}} \quad (6)$$

$$- j X_C = \frac{\bar{V}_{PC}}{\bar{I}_C} \quad (7)$$

$$x^2 - y^2 = (x+y)(x-y) \quad (1)$$

$$x^2 - y^2 = (x+y)(x-y) \quad (2)$$

$$x^2 - y^2 = (x+y)(x-y) \quad (3)$$

The above three equations are identical, and the only difference between them is the order of the terms. In the first equation, the terms are arranged in the order of their appearance in the original expression. In the second equation, the terms are arranged in the order of their appearance in the original expression, but the order of the terms is reversed. In the third equation, the terms are arranged in the order of their appearance in the original expression, but the order of the terms is reversed, and the terms are also grouped together.

### CONCLUSION

The above three equations are identical, and the only difference between them is the order of the terms. In the first equation, the terms are arranged in the order of their appearance in the original expression. In the second equation, the terms are arranged in the order of their appearance in the original expression, but the order of the terms is reversed. In the third equation, the terms are arranged in the order of their appearance in the original expression, but the order of the terms is reversed, and the terms are also grouped together.

$$(1) \quad x^2 - y^2 = (x+y)(x-y)$$

$$(2) \quad x^2 - y^2 = (x+y)(x-y)$$

$$(3) \quad x^2 - y^2 = (x+y)(x-y)$$

Figure X of Appendix B shows vector diagrams drawn for several load power factors. Figure III shows how  $X_{L1}$ ,  $X_{L2}$ , and  $X_G$  vary with load power factor.

### EXPERIMENTAL VERIFICATION OF GRAPHICAL STATIC ANALYSIS

The next step in the procedure was to verify experimentally the results of the graphical static analysis. Figure III compares calculated and measured values of the balancing reactances, and substantial agreement is indicated. Figure IV compares calculated and measured values of phase currents under balanced conditions.\* Calculated values are based upon Equation (4) which assumes no losses in the balancing unit. As was expected, the measured phase currents were larger than the calculated values due to some dissipation in the inductors.

### PRINCIPLE OF OPERATION OF AUTOMATIC BALANCING UNIT

With the accomplishment of these preliminary steps, the next objective was to design an automatic balancing unit. The fundamental principle upon which our design is based can be stated as follows:

If the values of the balancing reactances,

$X_{L1}$ ,  $X_{L2}$ , and  $X_G$ , are continuously adjusted so that the three phase currents are in phase with the line-to-line voltages, balanced phase currents will result.

By inductive reasoning, as set forth in detail in Appendix B, it can be shown that a shift in any of the three phase currents from its balanced position results in an unbalanced system; conversely, when the

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\* Data for Figures III and IV will be found in Tables I and II of Appendix D.

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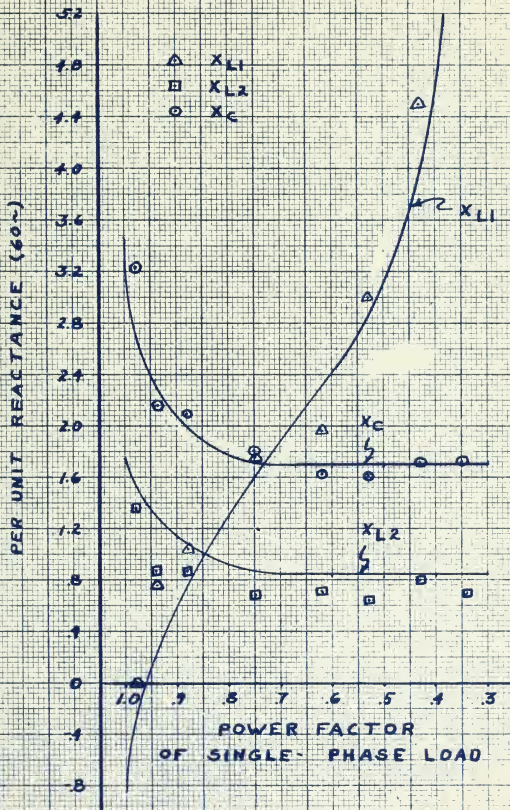
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FIGURE III

PER UNIT VALUES OF BALANCING REACTANCES

$$1.0 \text{ PER UNIT } X = \frac{V_L}{I_L}$$



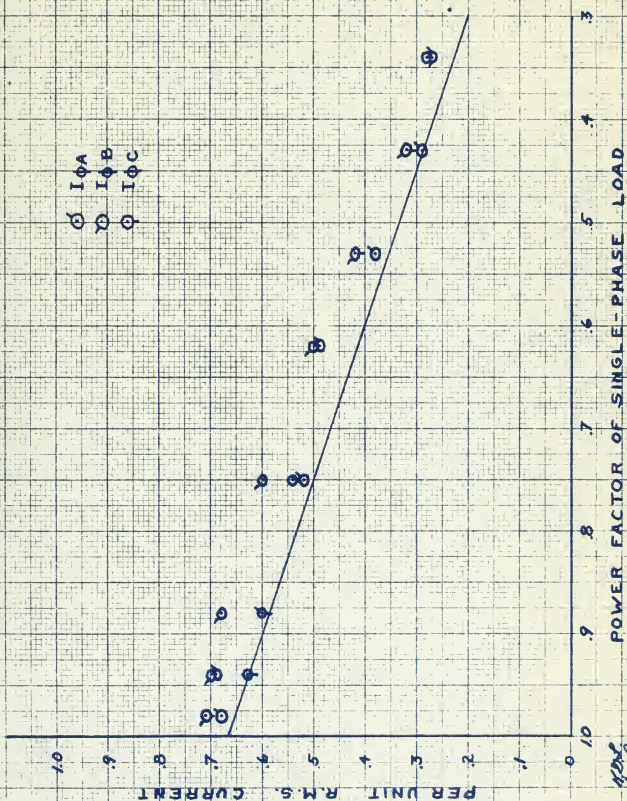
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FIGURE IV

PER UNIT PHASE CURRENTS VS. POWER FACTOR

10 PER UNIT CURRENT =  $I_L$



WPA  
SFA  
2-10-51



three phase currents are in phase with the line-to-line voltages, balanced currents result. This principle was verified experimentally when the data of Figures III and IV was obtained. The balancing reactances were adjusted successively until each phase current was in phase with its line-to-line voltage (as indicated by Lissajous Figures on a CRO screen), and it was noted that this process always produced a balanced system.

Stated simply, the problem of designing an automatic balancing unit now becomes a matter of making the balancing reactances change in response to phase differences between the phase currents and the line-to-line voltages. Figure V(a) shows in block diagram form how we propose to accomplish this. The differential and the block K combined represent an error sensitive element whose output,  $V_o$ , is a function of the phase difference between the phase current and the line-to-line voltage. Block  $\phi(X)$  represents a system which causes the balancing reactance,  $X$ , to change in accordance with the polarity of  $V_o$ . Finally, the balancing reactance operates to change the phase position of the phase current to complete the feedback loop.

How this system accomplishes its purpose can be explained by considering the factors which determine the position of the phase currents. Combining Equations (3) and (7) we have:

$$\bar{I}_{\phi C} = \bar{I}_L + \bar{I}_C = \bar{I}_L + \frac{\bar{V}_{PC}}{-jX_C}$$

Combining Equations (2) and (5) and noting that  $\bar{I}_C = \bar{I}_{C1} + \bar{I}_{C2}$

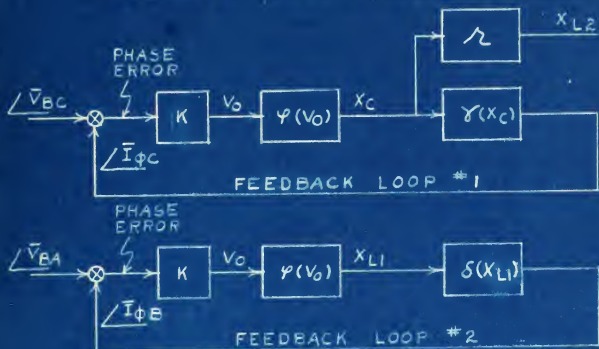
we have

$$\bar{I}_{\phi B} = \bar{I}_M - \bar{I}_L - \bar{I}_{C2} = (\bar{I}_M + \bar{I}_{C1}) - \bar{I}_L - \bar{I}_C = \frac{\bar{V}_{AP}}{jX_{L1}} - \bar{I}_L + \frac{\bar{V}_{PC}}{jX_C}$$

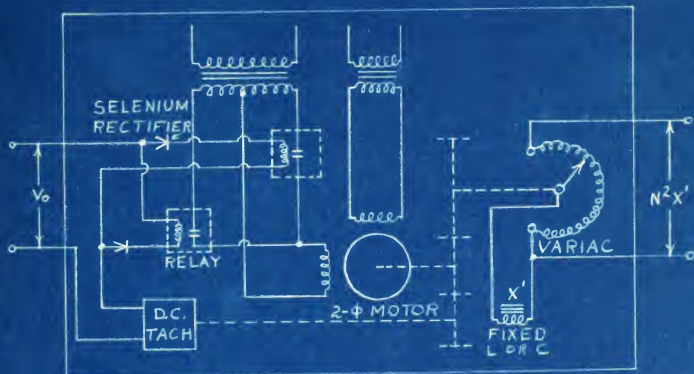


FIGURE V

DETAILS OF PROPOSED BALANCING UNIT



(A) SIMPLIFIED BLOCK DIAGRAM OF BALANCING UNIT.



(B) SCHEMATIC DIAGRAM OF  $\varphi(V_0)$  BLOCK IN (A).

Handwritten signature and date: 3-5-57



Since the position of point P in Figure II is a function of the value of all three balancing reactances, we can write:

$$\bar{I}_{\phi C} = \gamma (\bar{I}_L, X_{L1}, X_{L2}, X_C)$$

$$\bar{I}_{\phi B} = \delta (\bar{I}_L, X_{L1}, X_{L2}, X_C)$$

Considering Figure II in conjunction with the above equations it can be seen that  $X_C$  has the preponderant effect upon  $\bar{I}_{\phi C}$  and that by adjusting it alone, we can change  $\bar{I}_C$  to maintain  $\bar{I}_{\phi C}$  in phase with  $\bar{V}_{CB}$ . This is the function of Feedback Loop #1. Similarly,  $X_{L1}$  has the preponderant effect upon  $\bar{I}_{\phi B}$  and by adjusting it alone we can change  $\bar{I}_M$  to maintain  $\bar{I}_{\phi B}$  in phase with  $\bar{V}_{BA}$ . This is the function of Feedback Loop #2.

It should be noted that no feedback loop was employed to maintain  $\bar{I}_{\phi A}$  in phase with  $\bar{V}_{AC}$ . Referring to Figure III we can see that there is almost a constant ratio between  $X_{L2}$  and  $X_C$  in the range of load power factors between .3 and .8 and that this ratio varies only slightly at higher power factors. In consideration of this fact, it was decided to eliminate the third feedback loop by coupling the reactance  $X_{L2}$  directly to  $X_C$  by means of a constant gear ratio as shown in Figure V(a).

It should now become obvious that the system functions on the basis of a step-by-step process as the two feedback loops continuously correct  $X_{L1}$ ,  $X_{L2}$ , and  $X_C$  until balanced phase currents are obtained.

#### BALANCING UNIT COMPONENTS

The error sensitive element is a Differential Half-Wave Phase Detector whose output voltage,  $V_o$ , is a function of the phase angle,  $\phi$ ,

where the function  $f$  is defined by  $f(x) = \frac{1}{2}x^2$  and the function  $g$  is defined by  $g(x) = \frac{1}{2}x^2$ .

$$\begin{aligned} f(x) &= \frac{1}{2}x^2 \\ g(x) &= \frac{1}{2}x^2 \end{aligned}$$

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THE FUNCTION  $f(x) = \frac{1}{2}x^2$

The function  $f(x)$  is defined by  $f(x) = \frac{1}{2}x^2$  and the function  $g(x)$  is defined by  $g(x) = \frac{1}{2}x^2$ .

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existing between the two input voltages. A detailed explanation of this circuit is included in Appendix B. An R-C bridge network was employed to shift the line-to-line voltage by  $90^\circ$  before it was coupled to the phase detector reference input. A voltage proportional to the phase currents was obtained by inserting 3-ohm resistors in lines B and C and coupling the voltages across them to the phase detector signal input by means of a transformer with negligible magnetizing current.

Referring to Figure V(b), control of the 2-phase servo motor is achieved by means of a combination of relays and selenium rectifiers which control one field winding and make the motor turn in the proper direction in response to the polarity of  $V_o$ .

The method used to obtain a variable reactance was to reflect a fixed inductor or capacitor from the secondary winding of a Variac, which is the trade name for an adjustable auto-transformer. The contact arm of the Variac is driven by the servo motor through a step-down gear train. The fixed reactance is thus reflected to the output terminals of the Variac by the square of the variable turns ratio. In the preliminary investigations of this thesis, it was proposed to obtain variable inductance by means of a magnetic amplifier whose control winding would be excited by the phase detector output voltage. This scheme was subsequently abandoned because of excessive distortion in the output current of the magnetic amplifier.

A final point in this section concerns the matter of compensation. Tachometric feedback was employed to improve the stability of the system and provide more perfect follow-up. The tachometer was geared to the



shaft of the servo motor, and the voltage generated was subtracted from  $V_o$  as shown in Figure V(b).

#### DATA TAKEN ON AUTOMATIC BALANCING UNIT

The final step in the procedure was to record the static and dynamic performance of the balancing unit which was designed. Steady-state characteristics were obtained by recording for each phase current its magnitude, waveform, and the Lissajous Figure which resulted between the phase current and its line-to-line voltage. Transient response data was obtained by means of a bifilar oscillograph which recorded the transient nature of the three phase currents when the single-phase load was abruptly changed.



## RESULTS

The experimental work undertaken in this investigation confirmed the principle that an automatic balancing unit could be designed to reflect balanced three-phase currents from a single-phase load of variable magnitude and power factor.

### CIRCUIT DIAGRAM

Figure VI is a schematic diagram of the balancing unit. Figure XIV of Appendix D shows values and data for each item of equipment used.

### STEADY-STATE CHARACTERISTICS

Figure VII is a plot of the values of the three phase currents versus load power factor for balanced conditions. Table III of Appendix D gives the data for Figure VII. Table IV of Appendix D shows the current waveforms and corresponding Lissajous Figures for the steady-state characteristics of Figure VII.

### TRANSIENT RESPONSE

Figure VIII shows the transient response of the phase currents to (a) a step change from no-load to full-load and (b) a step change from full-load to no-load conditions.

Figure IX shows the transient response of the phase currents to (a) a step increase in load and (b) a step decrease in load.

## Section

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It is requested that the Commission be kept advised of any change in the

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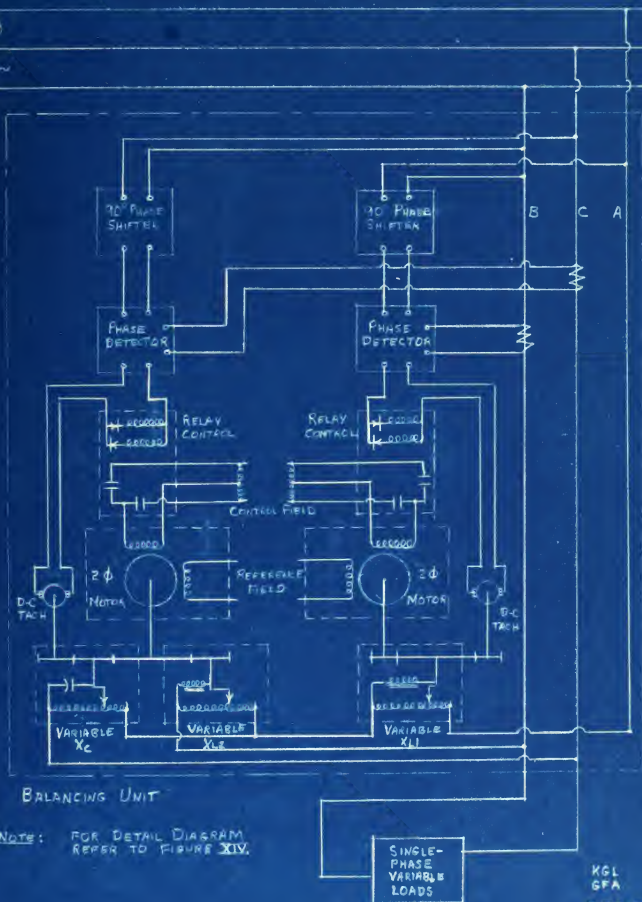
status of the case, and that the Commission be kept advised of any change in the

# FIGURE VI

## SCHEMATIC DIAGRAM OF BALANCING UNIT

3  $\phi$

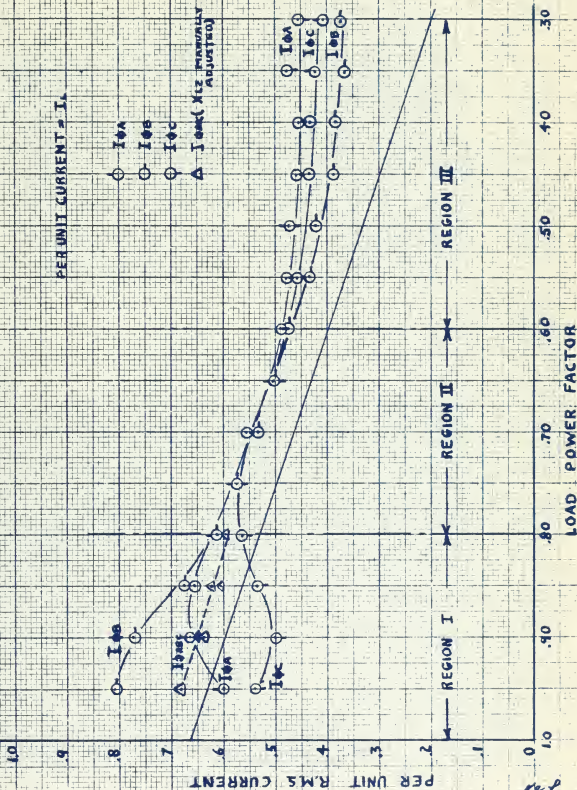
60~





# FIGURE VII

## STEADY-STATE CHARACTERISTICS

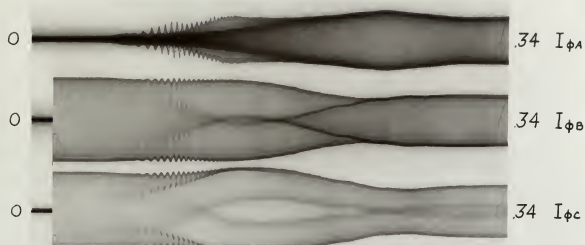


Red  
b7C



## FIGURE VIII

### TRANSIENT RESPONSE OF BALANCING UNIT



(A) NO-LOAD TO FULL-LOAD

← 14 SECONDS →



(B) FULL-LOAD TO NO-LOAD

NOTE: ACTUAL CURRENT VALUES  
ARE GIVEN IN AMPERES.

4-13-51  
H.B.L.  
J.F.R.



# FIGURE IX

## TRANSIENT RESPONSE OF BALANCING UNIT



(A) STEP INCREASE OF LOAD

← 14 SECONDS →



(B) STEP DECREASE OF LOAD

NOTE: ACTUAL CURRENT VALUES  
ARE GIVEN IN AMPERES.

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K.B.L.  
L.R.



## DISCUSSION OF RESULTS

### GENERAL

An analysis of the results will be made with two viewpoints in mind; (1) the power level and (2) the objective of the thesis. Primarily, the objective was to design and construct a network which would actually convert three-phase power into single-phase power, and secondly to design the network with a minimum of moving parts and vacuum tubes.

As stated in the results, the primary objective was fulfilled. In addition, power factors were improved to .866 in the three-phase system. The design of a similar unit to convert single-phase power to three-phase power was not attempted but may be pursued further.

### CIRCUIT DIAGRAM

The two main components of the balancing unit are (1) the error sensitive element and (2) the means of obtaining variable reactance (Fig. VI, Fig. XIV).

The phase detector was satisfactory in every respect. There were no moving parts in the phase shifter or phase detector, and the diode rectifiers could easily be replaced with selenium rectifiers. The twin-diode output current was of the order of 8 ma. More output current at this point would have loaded the signal voltage from the small resistor in the line. Greater output current from the detector could have been achieved using current transformers in the line and the selenium rectifiers mentioned previously.

Variable reactance was obtained by the use of a Variac which reflected its secondary impedance into the circuit by the square of the turns ratio.



The Variac is a relatively inexpensive, ruggedly constructed, variable auto-transformer with simple, direct action, requiring little maintenance. A small servo motor was used to physically position the Variac, and thereby introduced moving parts for the first time. The magnetic amplifier or saturable-core reactor discussed in the Procedure proved infeasible because of the large harmonic content in its output. Its possible further adaptation should not be ignored. Another possibility for achieving variable reactance without moving parts may be the use of a tap-changing transformer, the taps being actuated by relays in sequential order.

The servo motor used to position the Variac was a two-phase, 4-watt motor. Alternatively, a small d-c motor would have sufficed. The power for the two-phase motor was supplied by appropriate transformer arrangements from the three-phase lines. Since the error signal from the detector was d-c, it was necessary to control the a-c power of the two-phase motor by relays. (See Appendix C). A modulator and power amplifier driving one phase of the motor was considered but eliminated because of the introduction of vacuum tubes. It later proved necessary to bring vacuum tube amplifiers into the circuit since the current output of the detector was insufficient to actuate the relays used. The use of relays was continued, however, in the expectation that at higher power levels, the amplifiers would be unnecessary. If amplification is needed, magnetic amplifiers may be used to ensure the elimination of vacuum tubes.

From the nature of the feedback loop and the characterization of a positional servo with inertia,\* the motor output would be oscillatory. Either friction on the shaft or a voltage proportional to motor output

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\* References (7.) and (3.).



speed had to be fed back to damp the oscillation. The latter (tachometric feedback) was employed and enabled the use of much greater static gain in the system. A scheme for dynamic braking of the motor shaft in the relay dead zone is considered in Appendix C.

The success of the circuit as constructed is believed sufficient to warrant an engineering contract for the design and construction of a 4 to 10 KVA setup. Comparisons should be made with existing phase converter systems.

In this respect it may be noted that an increase in system frequency (i.e. 400 cycles) would reduce the size of the reactors by the ratio of the frequencies.

#### STEADY-STATE CHARACTERISTICS

Figure VII shows the steady-state line currents plotted against load power factors. The differences between the theoretical and observed balanced currents are due to three effects: (1) phase currents not exactly in phase with line-to-line voltages, (2) resistance losses in the inductors and losses in the Variac, and (3) harmonics in the currents due to non-linear elements or saturation. Items (1) and (3) have only a reactive power effect which is generally less than the original single-phase reactive load. The three regions of Figure VII corresponding to the above effects are indicated on the Graph as I, II, and III.

Region I is caused by the fact that  $\bar{I}_{\phi A}$  is not in phase with  $\bar{V}_{AC}$  as indicated in Table IV, Appendix D. In the design of the system,  $X_{L2}$  and  $X_C$  were assumed to have a constant ratio, and were thus coupled together. Figure XII(a) indicates a pronounced departure from this ratio



above load power factors of .8. Dashed curves on Figure VII were obtained by uncoupling  $X_{L2}$  and  $X_C$  and then manually adjusting  $X_{L2}$  to keep  $I_{\beta A}$  in phase with  $V_{AC}$  as indicated by a Lissajous Figure on the CRO. If loads are expected in the region of .8 to 1.0 power factor, three separate feedback circuits should be used to balance the currents.

Region II is what would normally be expected from such a unit, i.e. the difference between theoretical and observed values approximately equals the losses of the system. High-Q inductors can be easily made with higher efficiencies resulting. The brush drop of the Variac may be reduced with proper design. The use of relays to control the servo motor makes their idling losses nil, and the detector power losses can also be made negligible. An overall efficiency greater than 90% should be easily attained.

Region III is due to the saturation of  $X_{L1}$  which occurred at a power factor of about .6. The current rating in the secondary winding of the Variac has been exceeded and thus harmonics have been introduced into the phase currents as indicated in Table IV, Appendix D. Appendix C discusses in more detail the actual saturation of  $X_{L1}$  and how to properly design the elements to minimize non-linearities.

With three feedback loops adjusting  $X_{L1}$ ,  $X_{L2}$ , and  $X_C$  and with the proper design ratings of elements, results approximating the theoretical curve of Figure VII could be obtained for any operating region of load power factor.



### TRANSIENT RESPONSE:

Figures VIII and IX show the transient response obtained by applying step changes of load. The time scale and rms values of initial and final currents are indicated. The current deflection scales were not identical due to unequal line resistors and oscillograph gains.

The no-load currents indicated in Figure VIII could not be detected on the ammeters used, but they should not be more than the no-load loss of the three Variacs. This is given as 3 watts each (at 115 v) resulting in a total no-load loss of 5.2 watts (at  $\frac{115}{\sqrt{3}}$  v).

Transitory oscillations were of no importance since in every case balanced conditions were produced in less than 10 seconds. This response time is considered ample when compared with the nature of the expected load fluctuations. An improvement in response could be achieved with the use of dynamic braking as discussed in Appendix C.



## CONCLUSIONS AND RECOMMENDATIONS

### CONCLUSIONS

Based upon the results of this investigation, it is concluded that a balancing unit can be designed to accomplish automatic three-phase to single-phase power conversion with a minimum number of moving parts, no vacuum tubes, and without the use of rotating machinery. In addition, it may be possible to apply this scheme to the conversion of single-phase to three-phase power.

### RECOMMENDATIONS

The following is a list of recommended improvements in the design of an automatic balancing unit with a view toward possible engineering developments:

1. Selenium rectifiers should replace the vacuum tube rectifiers in the phase detector circuit.
2. A current transformer should be used to couple the line currents to the phase detector signal input.
3. A third feedback loop should be employed to adjust  $X_{L2}$  in response to the phase error of  $\bar{I}_{\phi A}$ .
4. Control of the two-phase motor may be improved by the schemes outlined in Appendix C.
5. Magnetic amplifiers may be used to provide any amplification required in a developed unit.
6. The possibility of using saturable-core reactors or tap-changing transformers to obtain variable reactances should be investigated.

In addition, it is recommended that a balancing unit of this type be adapted for single-phase to three-phase power conversion.



APPENDIX

## THEORY OF THE EARTH

The theory of the earth is a branch of geology which deals with the origin and development of the earth and its various parts. It is a science which seeks to explain the processes which have shaped the earth and its features. The theory of the earth is based on the study of the earth's history and its various parts. It is a science which seeks to explain the processes which have shaped the earth and its features. The theory of the earth is based on the study of the earth's history and its various parts.

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## APPENDIX A

### SUPPLEMENTARY INTRODUCTION

#### NOTE OF ERROR IN REFERENCES (4.) and (5.)

It is of interest to point out an error common to References (4.) and (5.) which will have an important significance to one interested in calculating values of  $X_{L1}$ ,  $X_{L2}$ , and  $X_C$  by means of Equations (5), (6), and (7). With reference to Figure II, both authors showed the locus of point P as a straight line joining A and B. This would be true if the only current through the reactor was the magnetizing current,  $\bar{I}_M$ . However, since the reactor current is made up of  $\bar{I}_M$  and portions of  $\bar{I}_C$ , the voltage across the reactors must lead the resultant current by  $90^\circ$  as shown in Figure II. This places the locus of point P outside the triangle ABC (see Figure X of Appendix E).  $\bar{V}_{AB}$  is thus the vector sum of  $\bar{V}_{AP}$  and  $\bar{V}_{PB}$ , the voltages across  $X_{L1}$  and  $X_{L2}$  respectively.

Failure to recognize this point would lead to large errors in calculating the values of reactances needed to obtain balanced conditions.

#### NAVAL APPLICATIONS

Although there have been some problems in connection with converting three-phase to single-phase power on naval surface vessels, they have been solved satisfactorily without the use of balancing equipment.\*

The type of balancing unit which we have designed would have an immediate application in fleet type submarines for converting single-phase to three-phase power. On SS 213-313 the only a-c power available is from three 15 KVA, 60 cycle, single-phase output motor-generators,

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\* References (10.) and (11.).

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while on SS 313 and up, this power is supplied by from four to six 12 KVA, 60-cycle, single-phase output motor-generator sets. Electronics and ordnance requirements are such that rotating machinery is being used to supply approximately 6 KVA, 120 volt, three-phase, 0.5 power factor loads in the first case and approximately 2 KVA, 120 volt, three-phase, 0.5 power factor loads in the latter.

On the latest class of submarines (SS 563), three-phase motor-generator sets supply both three-phase and single-phase loads. It may be that a phase balancing unit could be used advantageously to supply the single-phase load.

The Bureau of Ships has received several requests for information regarding three-phase to single-phase power conversion. One was in connection with a portable radio set in which a three-phase generator was used to supply both three-phase and single-phase power. Another inquiry concerned a school installation in which a fairly considerable block of single-phase power was needed for the operation of synchros.

and the fact that the Commission has not yet received any information from the Government of the United Kingdom regarding the results of its investigation into the matter. The Commission is therefore unable to make any further progress at this time.

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## APPENDIX B

### SUPPLEMENTARY PROCEDURE

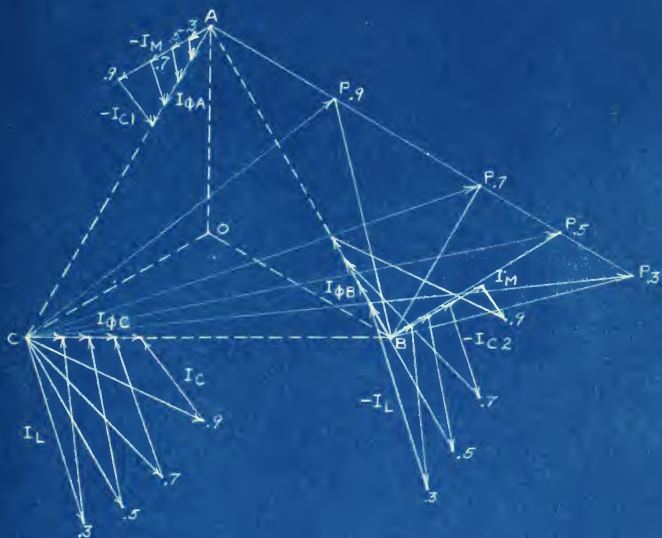
#### PROOF OF PRINCIPLE OF OPERATION OF AUTOMATIC BALANCING UNIT

In the Procedure it is stated that the automatic balancing unit was designed to function on the principle that a balanced three-phase load will be reflected to the source if the phase currents are maintained in phase with the line-to-line voltages. To prove this we must employ a process of inductive reasoning. Referring to Figure II, let us imagine the situation which would result if, with the system initially balanced,  $X_C$  was suddenly increased to make the magnitude of  $\bar{I}_C$  about half that shown. Since the current in line C must equal the vector sum of  $\bar{I}_L$  and  $\bar{I}_C$ ,  $\bar{I}_{\phi C}$  would now lag  $\bar{V}_{CB}$  by a small angle and become greater in magnitude than the vector shown. Assuming that the reactors were not changed, the vector  $\bar{I}_M$  would remain unchanged. Since  $\bar{I}_C$  is now smaller than before, both  $\bar{I}_{C1}$  and  $\bar{I}_{C2}$  will also become smaller in magnitude. It can now be seen that  $\bar{I}_{\phi B}$  will become greater in magnitude and slightly changed in direction due to the change in  $\bar{I}_{C2}$ . The magnitude of  $\bar{I}_{\phi A}$  will be slightly decreased and it will now lag  $\bar{V}_{AC}$  by an appreciable angle. Thus we see that all three currents,  $\bar{I}_{\phi A}$ ,  $\bar{I}_{\phi B}$ , and  $\bar{I}_{\phi C}$ , have become unbalanced in magnitude and phase due to shifting only  $\bar{I}_{\phi C}$  from its proper position. Applying a similar process of reasoning to a number of different assumptions leads to the conclusion stated previously that the system will be balanced only if the balancing reactances are adjusted so that each phase current is in phase with the proper line-to-line voltage.



FIGURE X

VECTOR DIAGRAMS FOR GRAPHICAL STATIC ANALYSIS



NOTE:

1.0 PER UNIT CURRENT =  $I_L$

1.0 PER UNIT VOLTAGE =  $V_{AC} = V_{CB} = V_{BA}$

*Handwritten:*  
LFA  
1-25-51



# DIFFERENTIAL HALF-WAVE PHASE DETECTOR

The circuit diagram and vector diagram of the phase detector are shown in Figure XI. The reference voltage must be shifted  $90^\circ$  to eliminate the ambiguity at  $\phi = 0$  and also to afford greater sensitivity. Thus it can be seen that if the signal and reference voltages are in phase ( $\phi = 0$ ), the vectors  $\bar{E}_1$  and  $\bar{E}_2$  will be equal in magnitude. By means of the smoothing R-C circuit, direct voltages approximately equal to the magnitudes of  $\bar{E}_1$  and  $\bar{E}_2$  will be produced at the output terminals. Since the polarities of these voltages are opposing, the output voltage,  $V_o$ , will be zero.

If there is a phase angle,  $\phi$ , between the signal and reference voltages,  $V_o$  will be related to  $\phi$  by the equation

$$V_o \approx \frac{2 \sqrt{2} E_a \sin \phi / 2}{\pi} *$$

which shows that  $V_o$  is a function of both  $E_a$  and  $\phi$ . In order to make the error voltage independent of the magnitude of the phase currents, the line-to-line voltage of the three-phase system was used as the reference voltage; and a voltage proportional to the phase current was introduced as the signal voltage.

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\* Reference (6.).

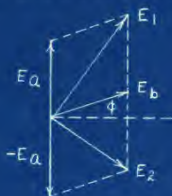


FIGURE XI

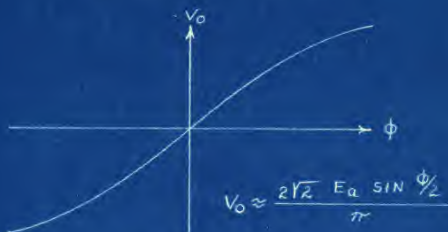
HALF-WAVE DIFFERENTIAL PHASE DETECTOR



(A) CIRCUIT DIAGRAM



(B) VECTOR DIAGRAM



(C) OUTPUT CHARACTERISTIC

NOTE: THIS FIGURE IS BASED  
ON REFERENCE (6.).

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J.S.L.  
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## APPENDIX C

### SUPPLEMENTARY DISCUSSION

#### SATURATION OF $X_{L1}$

To achieve proper ratings of the Variac, it is necessary to establish the range of power factors to be encountered, i.e. power factors a and b of Figure XII(B.). Then the coupled inductor on the Variac is  $x_a$  (for  $N = 1$ ) and the following relations hold:

$$x/x_a = N^2 \quad (8)$$

$$i_1 \times N = i_2 \quad (9)$$

To design the network in order to achieve as high a power factor as possible for balance,  $X_{L1}$  was purposely made low (60 ohms or .33 p.u. for  $I_L = .6$  amps). This permitted theoretical balance at a power factor of .95. At power factor = .6, however, the per unit  $X_{L1}$  required was 2.4; and with a base current of .6 amps, the secondary current was .73 amps (See Equations (8) and (9) above). This current is rapidly approaching the Variac rating of 1 amp and thus saturation occurred with harmonics being introduced (Table IV).

The maximum current ratings of the Variac can be designed, however, to prevent saturation over any range of load power factors. If complete range of power factors is to be reached, periodic paralleling of inductors (i.e. steps of  $x_b/x_a = 4$  or 9) would permit the current rating of the Variac to be of the order of two or three times the theoretical  $I_\phi$ , and thus materially reduce the required rating of the Variac on  $X_{L1}$ .

$X_{L2}$  and  $X_C$  have no wide range of reactances when  $I_L$  is constant, and they merely must be of the same order of current rating as the single-

The integral is a linear functional on the space of functions. It is linear in the sense that if  $f$  and  $g$  are functions and  $\alpha$  and  $\beta$  are constants, then

$$\int (\alpha f + \beta g) dx = \alpha \int f dx + \beta \int g dx$$

and it is linear in the sense that if  $f$  and  $g$  are functions and  $\alpha$  and  $\beta$  are constants, then

$$\int (\alpha f + \beta g) dx = \alpha \int f dx + \beta \int g dx$$

$$(i) \quad \int f dx = \int f dx$$

$$(ii) \quad \int f dx = \int f dx$$

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$$\int (\alpha f + \beta g) dx = \alpha \int f dx + \beta \int g dx$$

and it is linear in the sense that if  $f$  and  $g$  are functions and  $\alpha$  and  $\beta$  are constants, then

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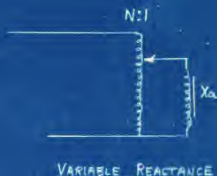
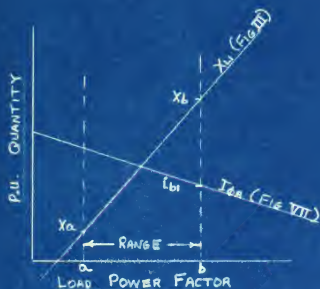
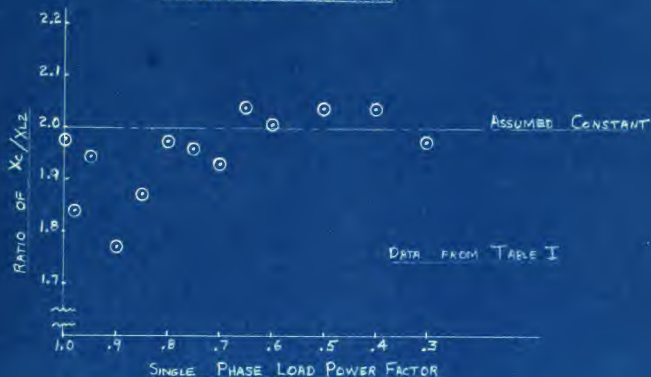
$$\int (\alpha f + \beta g) dx = \alpha \int f dx + \beta \int g dx$$

and it is linear in the sense that if  $f$  and  $g$  are functions and  $\alpha$  and  $\beta$  are constants, then

$$\int (\alpha f + \beta g) dx = \alpha \int f dx + \beta \int g dx$$

FIGURE XII

SATURATION OF  $X_{LI}$  VARIAC



FOR RANGE OF POWER FACTORS  $a$  to  $b$ :

$$N^2 X_2 = X_b$$

$$L_2 = I_{L1} N \approx \text{CURRENT RATING OF VARIAC}$$



phase load. With decrease in load current, the reactance varies as

$$\frac{I_{L \max}}{I_L} \text{ or } N \propto \sqrt{\frac{I_{L \max}}{I_L}}, \text{ and the secondary current then varies as } \frac{I_L}{I_{L \max}} \times \sqrt{\frac{I_{L \max}}{I_L}} \text{ which is always less than 1 for } I_L \text{ less than } I_{L \max}.$$

### TWO-PHASE MOTOR CONTROL

Three schemes of obtaining polarity sensitive motor control are shown in Figure XIII.

Scheme I indicates the method used in the balancing circuit as designed. If a signal has been applied and the motor is running, a decrease of the signal will permit the relay in use to open. Instead of coasting quickly to a stop, the motor continues to run due to its inertia and the single-phase torque of Field A. Sufficient damping was present so that this was not a serious objection, but schemes II and III show two alternate methods for handling this situation.

Scheme II consists of relay coils similar to scheme I, but the relays are double contact arrangements permitting opening and closing of the two fields simultaneously. The method is advantageous since it eliminates the single-phase driving torque and also the no-load loss in Field A.

Scheme III uses dynamic braking during the normally-open periods of relays #1 and #2 by the use of a third, normally-closed relay which shunts Field B. Relay #3 must in addition be set to open at a lower voltage than the normally-open relays #1 and #2 close, and conversely, relay #3 must close at a lower voltage than relays #1 and #2 open. No rectifier is provided in the coil of relay #3 since it must act as indicated above in response to a signal of either polarity.

is known that the function  $f(x)$  is continuous at  $x = a$  and that

$$\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} = L \quad (1)$$

$$\text{then } f(x) \text{ is differentiable at } x = a \text{ and the derivative is } f'(a) = L.$$

### THEOREM 1.1

Let  $f(x)$  be a function defined on an interval  $I$  containing  $a$ .

Then  $f(x)$  is differentiable at  $a$  if and only if

$$\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = L$$

and  $L$  is the derivative of  $f$  at  $a$ , denoted by  $f'(a)$ .

Proof. Suppose  $f(x)$  is differentiable at  $a$ . Then

$$\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} = L$$

and, by the definition of a limit, for any  $\epsilon > 0$  there is a  $\delta > 0$  such that

$$0 < |x - a| < \delta \implies \left| \frac{f(x) - f(a)}{x - a} - L \right| < \epsilon$$

Let  $h = x - a$ . Then  $0 < |h| < \delta$  implies

$$\left| \frac{f(a+h) - f(a)}{h} - L \right| < \epsilon$$

which is exactly the condition for  $f(x)$  to be differentiable at  $a$  with derivative  $L$ .

Conversely, suppose  $f(x)$  is differentiable at  $a$  with derivative  $L$ . Then

$$\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = L$$

and, by the definition of a limit, for any  $\epsilon > 0$  there is a  $\delta > 0$  such that

$$0 < |h| < \delta \implies \left| \frac{f(a+h) - f(a)}{h} - L \right| < \epsilon$$

Let  $x = a + h$ . Then  $0 < |x - a| < \delta$  implies

$$\left| \frac{f(x) - f(a)}{x - a} - L \right| < \epsilon$$

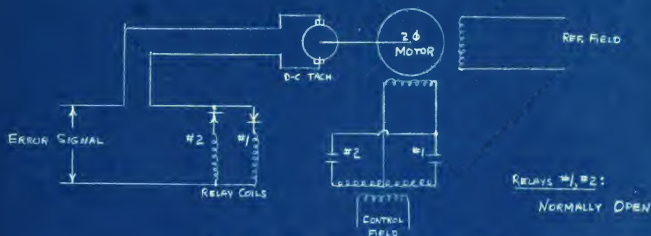
which is exactly the condition for  $f(x)$  to be differentiable at  $a$  with derivative  $L$ .

Thus,  $f(x)$  is differentiable at  $a$  if and only if

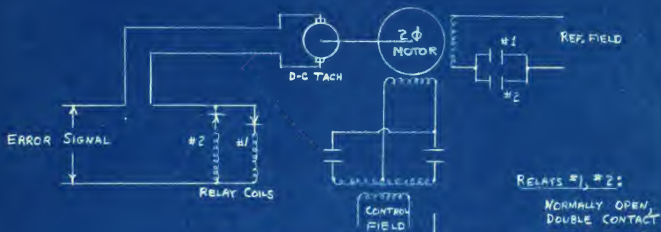
$$\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = L$$

and  $L$  is the derivative of  $f$  at  $a$ , denoted by  $f'(a)$ .

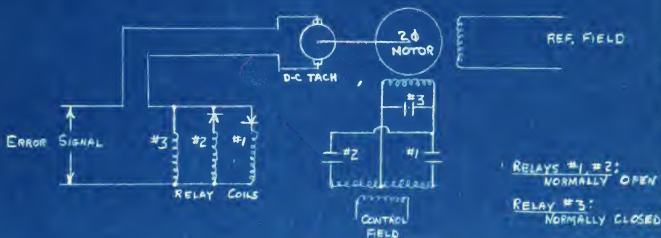
# FIGURE XIII TWO-PHASE MOTOR CONTROL



SCHEME I (ONE-FIELD CONTROL)



SCHEME II (TWO-FIELD CONTROL)



SCHEME III (DYNAMIC BRAKING)



APPENDIX D

DATA



TABLE I

DATA OBTAINED BY GRAPHICAL STATIC ANALYSIS

| <u>cos <math>\theta</math></u> | <u><math>E_{CP}</math></u> | <u><math>I_C</math></u> | <u><math>X_C</math></u> | <u><math>E_{PB}</math></u> | <u><math>\bar{I}_M - \bar{I}_{G2}</math></u> | <u><math>X_{L2}</math></u> | <u><math>E_{PA}</math></u> | <u><math>\bar{I}_M + \bar{I}_{G1}</math></u> | <u><math>X_{L1}</math></u> |
|--------------------------------|----------------------------|-------------------------|-------------------------|----------------------------|----------------------------------------------|----------------------------|----------------------------|----------------------------------------------|----------------------------|
| 1.00                           | 1.15                       | .333                    | 3.46                    | 1.52                       | .87                                          | 1.75                       | -.57                       | .667                                         | -.86                       |
| .95                            | 1.01                       | .44                     | 2.30                    | .80                        | .68                                          | 1.18                       | .21                        | .634                                         | .34                        |
| .90                            | 1.07                       | .535                    | 2.00                    | .68                        | .60                                          | 1.13                       | .39                        | .600                                         | .65                        |
| .85                            | 1.14                       | .60                     | 1.90                    | .545                       | .54                                          | 1.01                       | .59                        | .566                                         | 1.04                       |
| .80                            | 1.21                       | .667                    | 1.82                    | .52                        | .565                                         | .92                        | .69                        | .535                                         | 1.29                       |
| .75                            | 1.26                       | .71                     | 1.78                    | .48                        | .53                                          | .91                        | .78                        | .500                                         | 1.56                       |
| .70                            | 1.30                       | .765                    | 1.70                    | .48                        | .54                                          | .89                        | .80                        | .466                                         | 1.72                       |
| .65                            | 1.37                       | .795                    | 1.72                    | .48                        | .57                                          | .844                       | .96                        | .434                                         | 2.22                       |
| .60                            | 1.39                       | .83                     | 1.68                    | .50                        | .60                                          | .835                       | .98                        | .400                                         | 2.45                       |
| .50                            | 1.50                       | .89                     | 1.69                    | .55                        | .67                                          | .825                       | 1.12                       | .334                                         | 3.36                       |
| .40                            | 1.58                       | .93                     | 1.70                    | .61                        | .73                                          | .835                       | 1.20                       | .266                                         | 4.51                       |
| .30                            | 1.67                       | .97                     | 1.72                    | .69                        | .79                                          | .875                       | 1.35                       | .20                                          | 6.75                       |

**NOTE:**

(1) All quantities are per unit.

1.0 per unit current =  $I_L$

1.0 per unit voltage =  $V_{\text{line-to-line}}$

(2) See Figure II for notation.

(3) This data is plotted on Figure III (solid lines).

(4) Calculations were made using Equations (5), (6), and (7).

# TABLE

VALUES OF  $\log_{10} \Gamma(x)$  FOR  $x$  FROM 0.00 TO 1.00

| $x$  | $x^2$ | $x^3$ | $x^4$ | $x^5$ | $x^6$ | $x^7$ | $x^8$ | $x^9$ | $x^{10}$ |
|------|-------|-------|-------|-------|-------|-------|-------|-------|----------|
| 0.00 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000    |
| 0.01 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000    |
| 0.02 | 0.004 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000    |
| 0.03 | 0.009 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000    |
| 0.04 | 0.016 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000    |
| 0.05 | 0.025 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000    |
| 0.06 | 0.036 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000    |
| 0.07 | 0.049 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000    |
| 0.08 | 0.064 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000    |
| 0.09 | 0.081 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000    |
| 0.10 | 0.100 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000    |
| 0.11 | 0.121 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000    |
| 0.12 | 0.144 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000    |
| 0.13 | 0.169 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000    |
| 0.14 | 0.196 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000    |
| 0.15 | 0.225 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000    |
| 0.16 | 0.256 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000    |
| 0.17 | 0.289 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000    |
| 0.18 | 0.324 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000    |
| 0.19 | 0.361 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000    |
| 0.20 | 0.400 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000    |

1000

(1) The function  $\Gamma(x)$  is defined by

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$$

for  $x > 0$ . It is a continuous function of  $x$ .

(2) The function  $\Gamma(x)$  is a continuous function of  $x$ .

(3) The function  $\Gamma(x)$  is a continuous function of  $x$ .

(4) The function  $\Gamma(x)$  is a continuous function of  $x$ .

TABLE II

EXPERIMENTAL DATA TO VERIFY GRAPHICAL STATIC ANALYSIS

| cos $\theta$ | Per Unit    | Base           | Per Unit Current |           | Per Unit Current |              |              |
|--------------|-------------|----------------|------------------|-----------|------------------|--------------|--------------|
|              | $I_L$<br>ma | $V_L$<br>volts | $I_{XC}$         | $I_{XL2}$ | $I_{\phi A}$     | $I_{\phi B}$ | $I_{\phi C}$ |
| .34          | 179         | 109            | .97              | .74       | .27              | .27          | .28          |
| .43          | 250         | 108            | .96              | .70       | .29              | .32          | .32          |
| .53          | 235         | 108            | .85              | .60       | .38              | .42          | .42          |
| .62          | 150         | 109            | .80              | .55       | .49              | .49          | .50          |
| .75          | 182         | 108            | .69              | .50       | .52              | .60          | .54          |
| .88          | 204         | 109            | .57              | .55       | .60              | .68          | .60          |
| .937         | 150         | 108            | .55              | .64       | .69              | .70          | .63          |
| .98          | 124         | 112            | .31              | .74       | .68              | .71          | .71          |

| cos $\theta$ | Per Unit<br>$V_{CP}$ | Unit<br>$V_{BP}$ | Voltage<br>$V_{AP}$ | Per Unit<br>$X_C$ | Unit<br>$X_{L2}$ | Reactance<br>$X_{L1}$ |
|--------------|----------------------|------------------|---------------------|-------------------|------------------|-----------------------|
| .34          | 1.67                 | .514             | 1.25                | 1.72              | .70              | 4.6                   |
| .43          | 1.65                 | .57              | 1.32                | 1.72              | .81              | 4.5                   |
| .53          | 1.37                 | .39              | 1.14                | 1.61              | .65              | 3.0                   |
| .62          | 1.30                 | .39              | .96                 | 1.63              | .71              | 1.96                  |
| .75          | 1.25                 | .34              | .92                 | 1.81              | .68              | 1.77                  |
| .88          | 1.19                 | .48              | .62                 | 2.09              | .87              | 1.03                  |
| .937         | 1.18                 | .56              | .52                 | 2.15              | .87              | .75                   |
| .98          | 1.00                 | 1.00             | 0                   | 3.23              | 1.35             | 0                     |

NOTE:

- (1) Circuit Diagram: Same as Figure I, with ammeters in series with  $X_C$ ,  $X_{L2}$ , Load, and in phases A, B, and C.

## TABLE

MEANS AND STANDARD DEVIATIONS OF THE MEASUREMENTS

| MEASUREMENTS |           |           | MEASUREMENTS |           | MEASUREMENTS |           | S.D. |
|--------------|-----------|-----------|--------------|-----------|--------------|-----------|------|
| $\bar{X}$    | $\bar{Y}$ | $\bar{Z}$ | $\bar{X}$    | $\bar{Y}$ | $\bar{X}$    | $\bar{Y}$ |      |
| 10           | 70        | 70        | 10           | 70        | 10           | 70        | 10   |
| 20           | 80        | 80        | 20           | 80        | 20           | 80        | 20   |
| 30           | 90        | 90        | 30           | 90        | 30           | 90        | 30   |
| 40           | 100       | 100       | 40           | 100       | 40           | 100       | 40   |
| 50           | 110       | 110       | 50           | 110       | 50           | 110       | 50   |
| 60           | 120       | 120       | 60           | 120       | 60           | 120       | 60   |
| 70           | 130       | 130       | 70           | 130       | 70           | 130       | 70   |
| 80           | 140       | 140       | 80           | 140       | 80           | 140       | 80   |
| 90           | 150       | 150       | 90           | 150       | 90           | 150       | 90   |
| 100          | 160       | 160       | 100          | 160       | 100          | 160       | 100  |

| MEASUREMENTS |           |           | MEASUREMENTS |           | MEASUREMENTS |           | S.D. |
|--------------|-----------|-----------|--------------|-----------|--------------|-----------|------|
| $\bar{X}$    | $\bar{Y}$ | $\bar{Z}$ | $\bar{X}$    | $\bar{Y}$ | $\bar{X}$    | $\bar{Y}$ |      |
| 10           | 70        | 70        | 10           | 70        | 10           | 70        | 10   |
| 20           | 80        | 80        | 20           | 80        | 20           | 80        | 20   |
| 30           | 90        | 90        | 30           | 90        | 30           | 90        | 30   |
| 40           | 100       | 100       | 40           | 100       | 40           | 100       | 40   |
| 50           | 110       | 110       | 50           | 110       | 50           | 110       | 50   |
| 60           | 120       | 120       | 60           | 120       | 60           | 120       | 60   |
| 70           | 130       | 130       | 70           | 130       | 70           | 130       | 70   |
| 80           | 140       | 140       | 80           | 140       | 80           | 140       | 80   |
| 90           | 150       | 150       | 90           | 150       | 90           | 150       | 90   |
| 100          | 160       | 160       | 100          | 160       | 100          | 160       | 100  |

END

TABLE OF MEASUREMENTS OF THE MEASUREMENTS OF THE MEASUREMENTS

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- (2) Apparatus: 2 fixed inductors (.5h)  
2 variable powder core inductors  
    .1h to 1.0h (4.3 ohms/.1h)  
    1.0h to 10 h (46 ohms/h)  
1 decade capacitor (.01 to 10  $\mu$ f)  
6 AC ammeters (accuracy 3%)  
1 slide wire resistor (1000 ohms)  
C.R.O. and V.T.V.M.
- (3) Accuracy of phase measurement using C.R.O. was  $\pm 10\%$  at  
low power factors and  $\pm 1\%$  at high power factors
- (4) Accuracy of balance was affected by the fact that the powder  
core inductors could only be adjusted in 10% steps.

(30)  $f$  satisfies  $\text{test2} \Leftarrow \text{true}$  iff

if  $\text{test1}$  then  $\text{test2} \Leftarrow \text{true}$  else

if  $\text{test3}$  then  $\text{test2} \Leftarrow \text{true}$  else

if  $\text{test4}$  then  $\text{test2} \Leftarrow \text{true}$  else

if  $\text{test5}$  then  $\text{test2} \Leftarrow \text{true}$  else

if  $\text{test6}$  then  $\text{test2} \Leftarrow \text{true}$  else

if  $\text{test7}$  then  $\text{test2} \Leftarrow \text{true}$

if  $\text{test8}$  then  $\text{test2} \Leftarrow \text{true}$  else  $\text{test2} \Leftarrow \text{false}$  (31)

if  $\text{test9}$  then  $\text{test2} \Leftarrow \text{true}$  else  $\text{test2} \Leftarrow \text{false}$

if  $\text{test10}$  then  $\text{test2} \Leftarrow \text{true}$  else  $\text{test2} \Leftarrow \text{false}$  (32)

if  $\text{test11}$  then  $\text{test2} \Leftarrow \text{true}$  else  $\text{test2} \Leftarrow \text{false}$

TABLE III

STEADY-STATE CHARACTERISTICS OF DESIGNED NETWORK

| $\cos \phi$ | $I_L$ (r.m.s.)<br>(amps) (P.A.) |     | $I_{\phi A}$ (r.m.s.)<br>(amps) (P.A.) |     | $I_{\phi B}$ (r.m.s.)<br>(amps) (P.A.) |     | $I_{\phi C}$ (r.m.s.)<br>(amps) (P.A.) |     |
|-------------|---------------------------------|-----|----------------------------------------|-----|----------------------------------------|-----|----------------------------------------|-----|
| .95         | .55                             | 1.0 | .33                                    | .60 | .44                                    | .80 | .29                                    | .53 |
| .90         | .53                             | 1.0 | .35                                    | .66 | .41                                    | .77 | .26                                    | .49 |
| .85         | .60                             | 1.0 | .39                                    | .65 | .40                                    | .67 | .32                                    | .53 |
| .80         | .60                             | 1.0 | .36                                    | .60 | .37                                    | .62 | .34                                    | .56 |
| .75         | .60                             | 1.0 | .34                                    | .57 | .34                                    | .57 | .34                                    | .56 |
| .70         | .60                             | 1.0 | .32                                    | .53 | .32                                    | .53 | .33                                    | .55 |
| .65         | .60                             | 1.0 | .30                                    | .50 | .30                                    | .50 | .31                                    | .51 |
| .60         | .60                             | 1.0 | .28                                    | .47 | .28                                    | .47 | .29                                    | .48 |
| .55         | .60                             | 1.0 | .27                                    | .45 | .26                                    | .43 | .28                                    | .47 |
| .50         | .60                             | 1.0 | .28                                    | .47 | .25                                    | .42 | .28                                    | .47 |
| .45         | .60                             | 1.0 | .26                                    | .43 | .23                                    | .38 | .27                                    | .45 |
| .40         | .60                             | 1.0 | .26                                    | .43 | .23                                    | .38 | .27                                    | .45 |
| .35         | .60                             | 1.0 | .25                                    | .42 | .22                                    | .36 | .28                                    | .47 |
| .30         | .60                             | 1.0 | .24                                    | .40 | .23                                    | .38 | .27                                    | .45 |
| *.95        | .57                             | 1.0 | .39                                    | .68 | .39                                    | .68 | .38                                    | .67 |
| *.90        | .50                             | 1.0 | .33                                    | .66 | .33                                    | .66 | .32                                    | .64 |
| *.85        | .49                             | 1.0 | .31                                    | .63 | .31                                    | .63 | .30                                    | .61 |
| *.80        | .45                             | 1.0 | .28                                    | .62 | .28                                    | .62 | .27                                    | .60 |

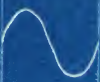
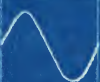
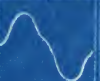
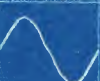
\* Note:  $X_{L2}$  disconnected from  $X_C$  and manually adjusted to keep  $\bar{I}_{\phi A}$  in phase with  $\bar{V}_{AC}$ .

## APPENDIX

## TABLE SHOWING THE RELATIONSHIP BETWEEN

| $(\text{Area})_1^2$<br>(sq. ft. (sq. ft.)) |      | $(\text{Area})_2^2$<br>(sq. ft. (sq. ft.)) |      | $(\text{Area})_3^2$<br>(sq. ft. (sq. ft.)) |      | $(\text{Area})_4^2$<br>(sq. ft. (sq. ft.)) |      | $\Delta$ (sq. ft.) |
|--------------------------------------------|------|--------------------------------------------|------|--------------------------------------------|------|--------------------------------------------|------|--------------------|
| 10.                                        | 10.  | 10.                                        | 10.  | 10.                                        | 10.  | 10.                                        | 10.  | 10.                |
| 20.                                        | 20.  | 20.                                        | 20.  | 20.                                        | 20.  | 20.                                        | 20.  | 20.                |
| 30.                                        | 30.  | 30.                                        | 30.  | 30.                                        | 30.  | 30.                                        | 30.  | 30.                |
| 40.                                        | 40.  | 40.                                        | 40.  | 40.                                        | 40.  | 40.                                        | 40.  | 40.                |
| 50.                                        | 50.  | 50.                                        | 50.  | 50.                                        | 50.  | 50.                                        | 50.  | 50.                |
| 60.                                        | 60.  | 60.                                        | 60.  | 60.                                        | 60.  | 60.                                        | 60.  | 60.                |
| 70.                                        | 70.  | 70.                                        | 70.  | 70.                                        | 70.  | 70.                                        | 70.  | 70.                |
| 80.                                        | 80.  | 80.                                        | 80.  | 80.                                        | 80.  | 80.                                        | 80.  | 80.                |
| 90.                                        | 90.  | 90.                                        | 90.  | 90.                                        | 90.  | 90.                                        | 90.  | 90.                |
| 100.                                       | 100. | 100.                                       | 100. | 100.                                       | 100. | 100.                                       | 100. | 100.               |
| 110.                                       | 110. | 110.                                       | 110. | 110.                                       | 110. | 110.                                       | 110. | 110.               |
| 120.                                       | 120. | 120.                                       | 120. | 120.                                       | 120. | 120.                                       | 120. | 120.               |
| 130.                                       | 130. | 130.                                       | 130. | 130.                                       | 130. | 130.                                       | 130. | 130.               |
| 140.                                       | 140. | 140.                                       | 140. | 140.                                       | 140. | 140.                                       | 140. | 140.               |
| 150.                                       | 150. | 150.                                       | 150. | 150.                                       | 150. | 150.                                       | 150. | 150.               |
| 160.                                       | 160. | 160.                                       | 160. | 160.                                       | 160. | 160.                                       | 160. | 160.               |
| 170.                                       | 170. | 170.                                       | 170. | 170.                                       | 170. | 170.                                       | 170. | 170.               |
| 180.                                       | 180. | 180.                                       | 180. | 180.                                       | 180. | 180.                                       | 180. | 180.               |
| 190.                                       | 190. | 190.                                       | 190. | 190.                                       | 190. | 190.                                       | 190. | 190.               |
| 200.                                       | 200. | 200.                                       | 200. | 200.                                       | 200. | 200.                                       | 200. | 200.               |
| 210.                                       | 210. | 210.                                       | 210. | 210.                                       | 210. | 210.                                       | 210. | 210.               |
| 220.                                       | 220. | 220.                                       | 220. | 220.                                       | 220. | 220.                                       | 220. | 220.               |
| 230.                                       | 230. | 230.                                       | 230. | 230.                                       | 230. | 230.                                       | 230. | 230.               |
| 240.                                       | 240. | 240.                                       | 240. | 240.                                       | 240. | 240.                                       | 240. | 240.               |
| 250.                                       | 250. | 250.                                       | 250. | 250.                                       | 250. | 250.                                       | 250. | 250.               |
| 260.                                       | 260. | 260.                                       | 260. | 260.                                       | 260. | 260.                                       | 260. | 260.               |
| 270.                                       | 270. | 270.                                       | 270. | 270.                                       | 270. | 270.                                       | 270. | 270.               |
| 280.                                       | 280. | 280.                                       | 280. | 280.                                       | 280. | 280.                                       | 280. | 280.               |
| 290.                                       | 290. | 290.                                       | 290. | 290.                                       | 290. | 290.                                       | 290. | 290.               |
| 300.                                       | 300. | 300.                                       | 300. | 300.                                       | 300. | 300.                                       | 300. | 300.               |
| 310.                                       | 310. | 310.                                       | 310. | 310.                                       | 310. | 310.                                       | 310. | 310.               |
| 320.                                       | 320. | 320.                                       | 320. | 320.                                       | 320. | 320.                                       | 320. | 320.               |
| 330.                                       | 330. | 330.                                       | 330. | 330.                                       | 330. | 330.                                       | 330. | 330.               |
| 340.                                       | 340. | 340.                                       | 340. | 340.                                       | 340. | 340.                                       | 340. | 340.               |
| 350.                                       | 350. | 350.                                       | 350. | 350.                                       | 350. | 350.                                       | 350. | 350.               |
| 360.                                       | 360. | 360.                                       | 360. | 360.                                       | 360. | 360.                                       | 360. | 360.               |
| 370.                                       | 370. | 370.                                       | 370. | 370.                                       | 370. | 370.                                       | 370. | 370.               |
| 380.                                       | 380. | 380.                                       | 380. | 380.                                       | 380. | 380.                                       | 380. | 380.               |
| 390.                                       | 390. | 390.                                       | 390. | 390.                                       | 390. | 390.                                       | 390. | 390.               |
| 400.                                       | 400. | 400.                                       | 400. | 400.                                       | 400. | 400.                                       | 400. | 400.               |
| 410.                                       | 410. | 410.                                       | 410. | 410.                                       | 410. | 410.                                       | 410. | 410.               |
| 420.                                       | 420. | 420.                                       | 420. | 420.                                       | 420. | 420.                                       | 420. | 420.               |
| 430.                                       | 430. | 430.                                       | 430. | 430.                                       | 430. | 430.                                       | 430. | 430.               |
| 440.                                       | 440. | 440.                                       | 440. | 440.                                       | 440. | 440.                                       | 440. | 440.               |
| 450.                                       | 450. | 450.                                       | 450. | 450.                                       | 450. | 450.                                       | 450. | 450.               |
| 460.                                       | 460. | 460.                                       | 460. | 460.                                       | 460. | 460.                                       | 460. | 460.               |
| 470.                                       | 470. | 470.                                       | 470. | 470.                                       | 470. | 470.                                       | 470. | 470.               |
| 480.                                       | 480. | 480.                                       | 480. | 480.                                       | 480. | 480.                                       | 480. | 480.               |
| 490.                                       | 490. | 490.                                       | 490. | 490.                                       | 490. | 490.                                       | 490. | 490.               |
| 500.                                       | 500. | 500.                                       | 500. | 500.                                       | 500. | 500.                                       | 500. | 500.               |

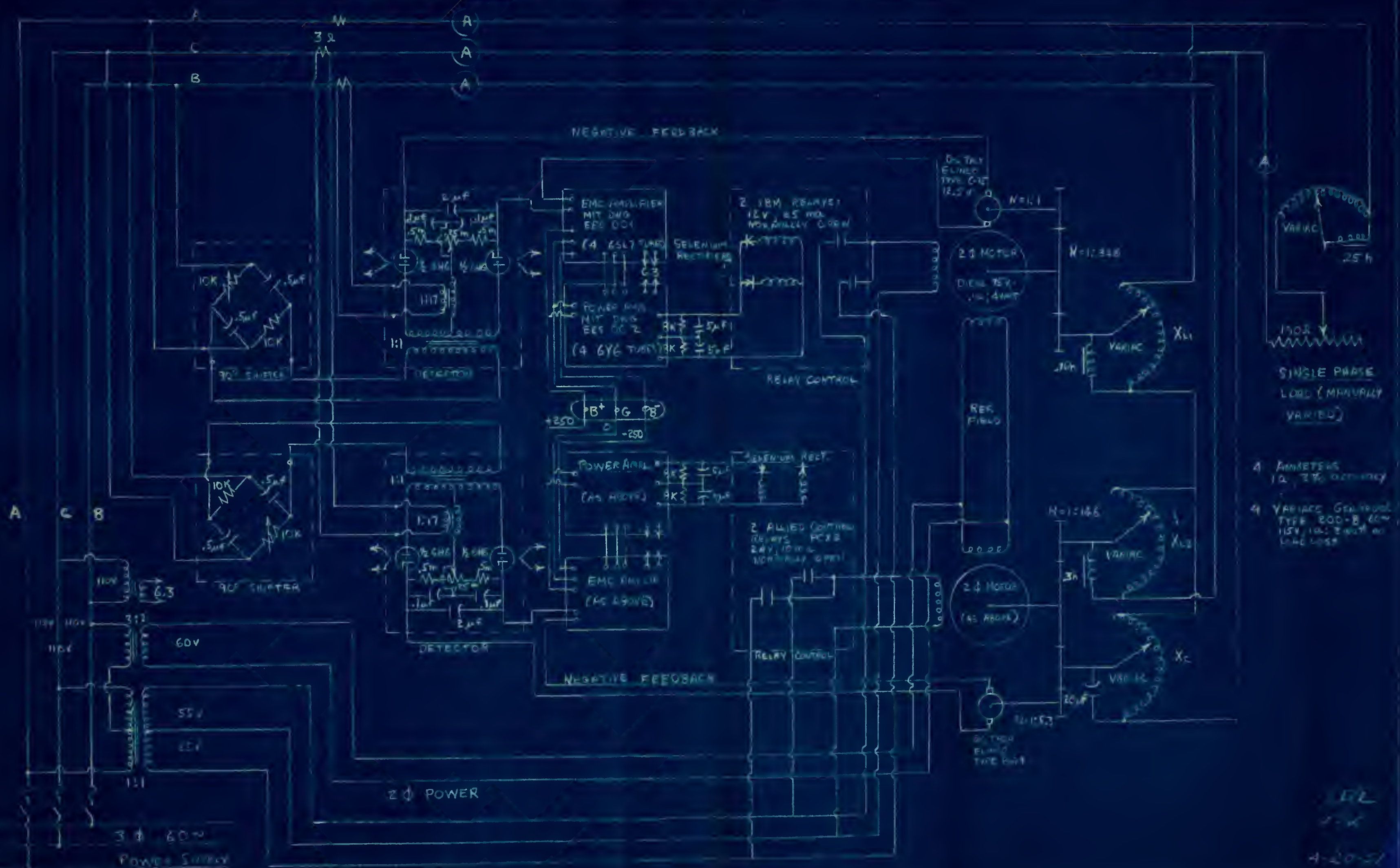
**TABLE IV**  
**LISSAJOUS AND LINE CURRENT WAVEFORMS OF TABLE III**

| LOAD<br>POWER<br>FACTOR        | $I \phi A$                                                                          |                                                                                     | $I \phi B$                                                                          |                                                                                     | $I \phi C$                                                                          |                                                                                     |                                                                                                                          |
|--------------------------------|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------|
|                                | LISSAJOUS                                                                           | CURRENT                                                                             | LISSAJOUS                                                                           | CURRENT                                                                             | LISSAJOUS                                                                           | CURRENT                                                                             |                                                                                                                          |
| .35                            |    |    |    |    |    |    |                                                                                                                          |
| .45                            |    |    |    |    |    |    |                                                                                                                          |
| .55                            |    |    |    |    |    |    |                                                                                                                          |
| .65                            |    |    |    |    |    |    |                                                                                                                          |
| .75                            |    |    |    |    |    |    |                                                                                                                          |
| .85                            |    |    |    |    |    |    |                                                                                                                          |
| .95                            |  |  |  |  |  |  |                                                                                                                          |
| XL2<br>MANUAL<br>ADJUST<br>.85 |  |  |  |  |  |  |                                                                                                                          |
| XL2<br>MANUAL<br>ADJUST<br>.95 |  |  |  |  |  |  | <div data-bbox="968 1317 1030 1390"> <i>hca</i><br/> <i>KCL</i> </div> <div data-bbox="968 1390 1030 1419">4-12-51</div> |



# FIGURE XIV

## DETAIL DIAGRAM OF BALANCING UNIT AS CONSTRUCTED





## APPENDIX E

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